
Queuing Analysis with QueueCalc

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QueueCalc Overview

- QueueCalc is a spreadsheet model for computing *steady state* (long term) estimates of queue performance.
- The basic inputs are the *arrival rate*, the *service rate per server*, and the *number of servers*. Note, that the arrival rate must be less than the service rate times the number of servers. If not, the queue length would continue to grow to infinity over the long term.
- The following slides show basic use of the model under two situations:
 - An assumption of maximally random arrival times and service times, appropriate for walk-in situations with high service variation
 - Less than maximal variation in either the arrival times or service times, appropriate if arrivals are somewhat regular or service times tend fall in a relatively narrow range.

A Simple Queue

Server

Person (MD, nurse, tech, transporter, housekeeper, etc.) or *Resource* (bed, scanner, equipment, etc.)

A Simple Queue

**Customer
Arrivals**

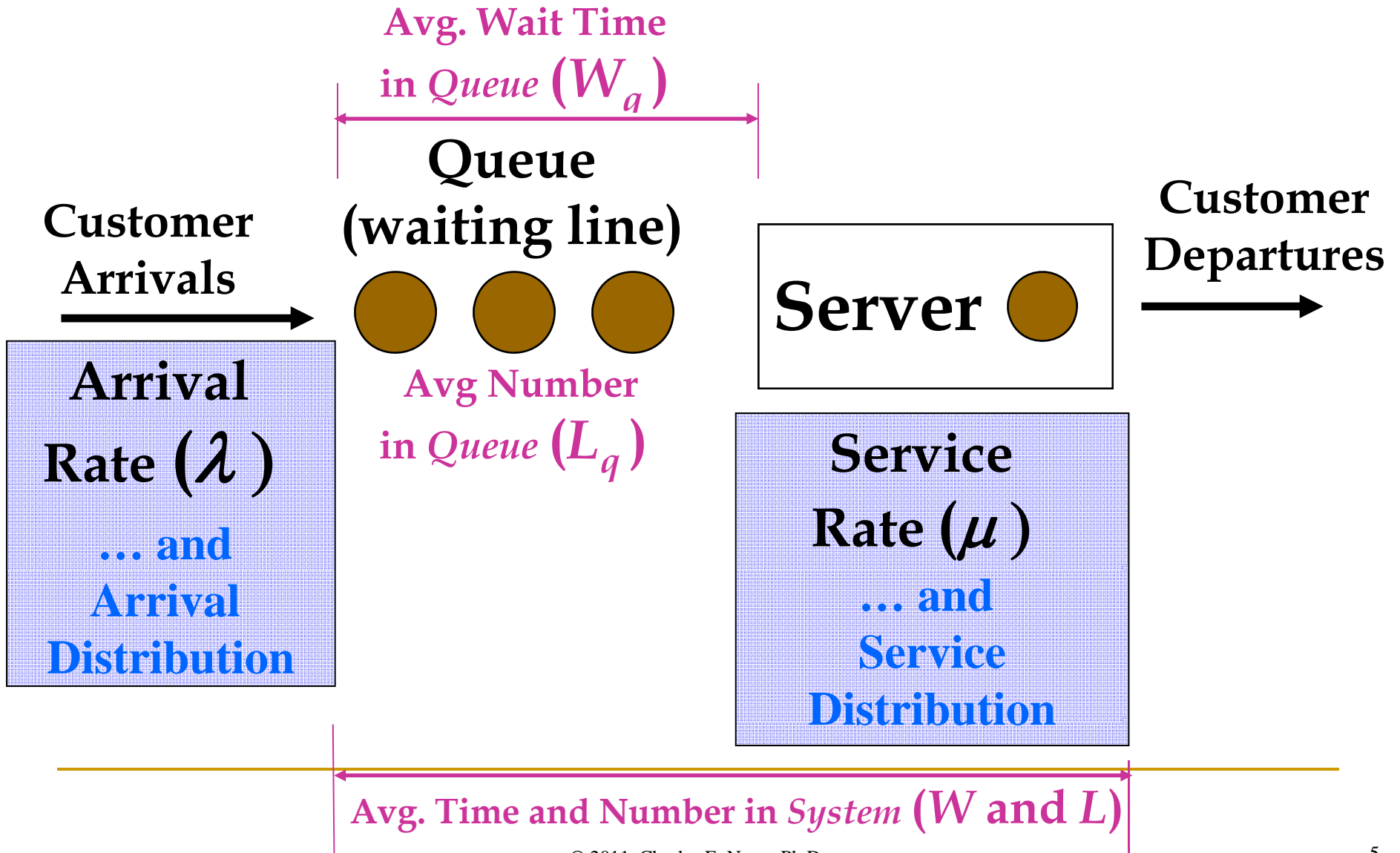


*Person (patient, call, etc.) or
Thing (lab sample, soiled
room, tray to be picked, etc.)*

Server

A rectangular box with a black border containing the word 'Server' in bold black text.

A Simple Queue



Walk-in Clinic Example

In a walk-in clinic, we have an average arrival rate of 4/hour served by a single doctor with an average service rate of 5/hour (average treatment time of 12 minutes). Assuming *maximal variation*, we can use **QueueCalc** to estimate steady-state values for:

1. Server Utilization _____
2. Average Time in the System (W) _____
3. Average Number of People in the System (L) _____
4. Average Wait Time (Wq) _____
5. Average Line Length (Lq) _____

QueueCalcSimple.xls [Compatibility Mode] - Microsoft Excel

Approximate Formula for Steady-State, Infinite Capacity Queues

INPUTS

Basic Inputs:

Number of Servers, $S =$	1	Average Time Between Arrivals 0.250
Arrival Rate, $\lambda =$	4	Average Service Time = 0.200
Service Rate Capacity of each server, $\mu =$	5	
Coefficient of Variation of Inter-arrival time, $CV(a) =$	1	
Coefficient of Variation of Service time, $CV(s) =$	1	

With the Coefficient of Variations set to 1, we are assuming maximal variation with Poisson arrivals and Exponentially distributed service times. Values of 0 would imply no variation.

Basic Outputs:

The Waiting Line:

Average Number Waiting in Queue (L_q) =	3.200	<== The Approximation
Average Waiting Time (W_q) =	0.8	

Service:

Average Utilization of Servers (ρ) =	80.00%
Average Number of Customers Receiving Service =	0.8

The Total System (waiting line plus customers being served):

Average Number in the System (L) =	4.000
Average Time in System (W) =	1

KEY OUTPUTS

Infinite Queue Approximation Worksheet

start | Infinite Queue Approximation | Poisson Distribution | 260% | 6:15 PM

Walk-in Clinic Example

In a walk-in clinic, we have an average arrival rate of 4/hour served by a single doctor with an average service rate of 5/hour (average treatment time of 12 minutes). Assuming *maximal variation*, we can use **QueueCalc** to estimate steady-state values for:

1. Server Utilization __80%__
2. Average Time in the System (W) __1 hour__
3. Average Number of People in the System (L) _4_
4. Average Wait Time (Wq) __.8 hour__
5. Average Line Length (Lq) _3.2_

Queuing Challenge

The walk-in clinic is about to launch an advertising campaign that is expected to boost demand by 15%. What implications will this have for the clinic? In particular, estimate :

0. The new arrival rate _____
1. Server Utilization _____
2. Average Time in the System (W) _____
3. Average Number of People in the System (L) _____
4. Average Wait Time (W_q) _____
5. Average Line Length (L_q) _____

QueueCalcSimple.xls [Compatibility Mode] - Microsoft Excel

Approximate Formula for Steady-State, Infinite Capacity Queues

Basic Inputs:

Number of Servers, S =	1	
Arrival Rate, λ =	4.6	Average Time Between Arrivals 0.217
Service Rate Capacity of each server, μ =	5	Average Service Time = 0.200
Coefficient of Variation of Inter-arrival time, $CV(a)$ =	1	
Coefficient of Variation of Service time, $CV(s)$ =	1	

Basic Outputs:

The Waiting Line: Average Number Waiting in Queue (L_q) = **10.580** <== The Approximation

Average Waiting Time (W_q) = 2.3

Service: Average Utilization of Servers (ρ) = **92.00%**
Average Number of Customers Receiving Service = **0.92**

The Total System (waiting line plus customers being served):

Average Number in the System (L) = **11.500**
Average Time in System (W) = **2.5**

Instructions Infinite Queue Approximation Poisson Distribution

Ready

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Queuing Challenge

The walk-in clinic is about to launch an advertising campaign that is expected to boost demand by 15%. What implications will this have for the clinic? In particular, estimate :

0. The new arrival rate 4.6
1. Server Utilization 92%
2. Average Time in the System (W) 2.5 hours
3. Average Number of People in the System (L) 11.5
4. Average Wait Time (Wq) 2.3 hours
5. Average Line Length (Lq) 10.58

Using QueueCalc for Systems that are not at Maximal Variation

- The *Infinite Queue Approximation* worksheet can be used to model queues in which arrival distributions are not entirely random and service distributions are not necessarily Exponential.
- For example, you would use it if your arrivals are a combination of scheduled and walk-in, or if your service times are for a relatively narrow set of activities.
- The following slides show how to analyze data and compute inputs for the Infinite Queue Approximation worksheet.

General Idea

- Characterize the Arrival Process
 - Gather data on arrival times
 - Compute inter-arrival times
 - Compute average, standard deviation, coefficient and coefficient of variation

 - Characterize the Service Process

 - Use QueueCalc to estimate the performance of the queuing interface
-

Modeling Queue Performance

Collect Data on:

1. Times of arrival
2. Service times

Compute:

- a. Average of the “time between arrivals”
- b. Standard Deviation of “time between arrivals”
- c. Average time of service
- d. Standard Deviation of time of service
- e. Average rate of arrivals ($1/a$)
- f. Coefficient of Variation of Arrivals (b/a)
- g. Average rate of service ($1/c$)
- h. Coefficient of Variation of Service (d/c)

**Inputs to
QueueCalc**

Microsoft Excel - Triage Queuing Analysis Simple.xls

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Arial 12

K30

	A	B	C	D	E	F	G	H	I	J	K
	Arrival Times	Inter-Arrival Times									
1	7										
2	28	21									
3	63										
4	119										
5	161										
6	182	21									
7	183	1									
8	188	5									
9	192	4									
10	196	4									
11	247	51									
12	251	4									
13	289	38									
14	301	12									
15	313	12									
16	322	9									
17	327	5									
18	361	34									
19	379	18									
20	387	8									
21	395	8									
22	403	8									
23	414	11									
24	422	8									
25	427	5									
26	475	48									
27											
28		19	17.011	3.25	0.908711262						
29		Average	Std.Dev.	Rate	Coeff. of Var						

List of Arrival Times over some period.

For example, these are for an 8-hour period from 8AM-4PM. The first arrival was at 8:07, the second at 8:28, the third at 9:03, and so on.

Example 1 / Example 2

Ready

Microsoft Excel - Triage Queuing Analysis Simple.xls

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Arial 12

K30

	A	B	C	D	E	F	G	H	I	J	K
	Arrival Times	Inter-Arrival Times									
1											
2	7										
3	28	21									
4	63	35									
5	119	56									
6	161	42									
7	182	21									
8	183	1									
9	188	5									
10	192	4									
11	196	4									
12	247	51									
13	251	4									
14	289	38									
15	301	12									
16	313	12									
17	322	9									
18	327	5									
19	361	34									
20	379	18									
21	387	8									
22	395	8									
23	403	8									
24	414	11									
25	422	8									
26	427	5									
27	475	48									
28		19	17.011	3.25	0.908711262						
29		Average	Std.Dev.	Rate	Coeff. of Var						

Inter-Arrival Times computed (for example, 21=28-7)

Example 1 / Example 2

Ready

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Arial 12

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	A	B	C	D	E	F	G	H	I	J	K
	Arrival Times	Inter-Arrival Times									
1											
2	7										
3	28	21									
4	63	35									
5	119	56									
6	161	42									
7	182	21									
8	183	1									
9	188	5									
10	192	4									
11	196	4									
12	247	51									
13	251	4									
14	289	38									
15	301	12									
16	313	12									
17	322	9									
18	327	5									
19	361	34									
20	379	18									
21	387	8									
22	395	8									
23	403	8									
24	414	11									
25	422	8									
26	427	5									
27	475	48									
28		19	17.011	3.25	0.908711262						
29		Average	Std.Dev.	Rate	Coeff. of Var						

Average of Inter-Arrival Times
“= average(B3..B27)”

Standard Deviation of Inter-Arrival Times
“= stdev(B3..B27)”

Arrival Rate per hour simply computed as 26 arrivals divided by 8

Coefficient of Variation computed as Standard Deviation divided by Average

Example 1 / Example 2

Ready

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Reply with Changes...

Arial 12

K30

	A	B	C	D	E	F	G	H	I	J	K
	Arrival Times	Inter-Arrival Times					Triage Times				
1											
2	7						5				
3	28	21					10				
4	63	35					23				
5	119	56					25				
6	161	42					25				
7	182	21					25				
8	183	1					8				
9	188	5					5				
10	192	4					18				
11	196	4					10				
12	247	51					15				
13	251	4					25				
14	289	38					13				
15	301	12					20				
16	313	12					15				
17	322	9					5				
18	327	5					8				
19	361	34					25				
20	379	18					25				
21	387	8					23				
22	395	8					25				
23	403	8					8				
24	414	11					13				
25	422	8					20				
26	427	5					5				
27	475	48					15				
28		19	17.011	3.25	0.908711262		16	7.667714533	3.7681	0.481547289	
29		Average	Std.Dev.	Rate	Coeff. of Var		Average	Std.Dev.	Rate	Coeff. of Var	

Obtain Data on Service Times then compute similar statistics. Assume these times represent the actual time spent from the start of an encounter until the time that an immediately following encounter could be started.

Average of Service Times
“= average(G2..G27)”

Standard Deviation of Service Times
“= stdev(G2..G27)”

Service Rate per hour computed as 60 minutes divided by Average Service Time

Coeff. of Var. as before

Example 1 / Example 2

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Microsoft Excel - Triage Queuing Analysis Simple.xls

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Reply with Changes...

Arial 12

K30

	A	B	C	D	E	F	G	H	I	J	K
	Arrival Times	Inter-Arrival Times					Triage Times				
1											
2	7						5				
3	28	21					10				
4	63	35					23				
5	119	56					25				
6	161	42					25				
7	182	21					25				
8	183	1					8				
9	188	5					5				
10	192	4					18				
11	196	4					10				
12	247	51					15				
13	251	4					25				
14	289	38					13				
15	301	12					20				
16	313	12					15				
17	322	9					5				
18	327	5					8				
19	361	34					25				
20	379	18					25				
21	387	8					23				
22	395	8					25				
23	403	8					8				
24	414	11					13				
25	422	8					20				
26	427	5					5				
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“= average(G2..G27)”

Standard Deviation of Service Times
“= stdev(G2..G27)”

Service Rate per hour computed as 60 minutes divided by Average Service Time

Coeff. of Var. as before

Inputs to QueueCalc

QueueCalcSimple.xls [Compatibility Mode] - Microsoft Excel

Approximate Formula for Steady-State, Infinite Capacity Queues

Basic Inputs:

Number of Servers, $S =$	1	
Arrival Rate, $\lambda =$	3.25	Average Time Between Arrivals 0.308
Service Rate Capacity of each server, $\mu =$	3.77	Average Service Time = 0.265
Coefficient of Variation of Inter-arrival time, $CV(a) =$	0.91	
Coefficient of Variation of Service time, $CV(s) =$	0.48	

Basic Outputs:

The Waiting Line: Average Number Waiting in Queue (L_q) = **2.852** **<== The Approximation**

Average Waiting Time (W_q) = 0.88

Service: Average Utilization of Servers (ρ) = **86.21%**
Average Number of Customers Receiving Service = **0.86207**

The Total System (waiting line plus customers being served):

Average Number in the System (L) = **3.714**
Average Time in System (W) = **1.14266**

Instructions Infinite Queue Approximation Poisson Distribution

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Now Try it on 2 Hours of Data

- Characterize the Arrival Process
 - Gather data on arrival times
 - Compute inter-arrival times
 - Compute average, standard deviation, coefficient and coefficient of variation

 - Characterize the Service Process

 - Use QueueCalc to estimate the performance of the queuing interface
-

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Reply with Changes... End Review...

Arial 10

K50

	A	B	C	D	E	F	G	H	I	J	K	L
	Arrival Times						Triage Times					
1												
2	606						9					
3	610						6					
4	617						7					
5	617						6					
6	627						4					
7	629						6					
8	636						4					
9	648						2					
10	653						8					
11	657						6					
12	658						6					
13	677						7					
14	699						4					
15	704						4					
16	712						4					
17	719						2					
18												
19												
20												
21												
22												
23												
24												
25												
26												
27												
28												

Sheet1 (2) Sheet1 Sheet2 Sheet3

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The DATA

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Reply with Changes... End Review...

Arial 10

K30

	A	B	C	D	E	F	G	H	I	J	K	L
	Arrival Times	Inter-Arrival Times					Triage Times					
1	606						9					
2	610	4					6					
3	617	7					7					
4	617	0					6					
5	627	10					4					
6	629	2					6					
7	636	7					4					
8	648	12					2					
9	653	5					8					
10	657	4					6					
11	658	1					6					
12	677	19					7					
13	699	22					4					
14	704	5					4					
15	712	8					4					
16	719	7					2					
17												
18		8	6.186	8	0.821151201			5	1.9906	11.294	0.37470173	
19		Average	Std.Dev.	Rate	Coeff. of Var			Average	Std.Dev.	Rate	Coeff. of Var	
20												
21												
22												
23												
24												
25												
26												
27												
28												

After DATA ANALYSIS

Sheet1 (2) Sheet1 Sheet2 Sheet3

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QueueCalcSimple.xls [Compatibility Mode] - Microsoft Excel

Approximate Formula for Steady-State, Infinite Capacity Queues

Basic Inputs:

Number of Servers, $S = 1$

Arrival Rate, $\lambda = 8$ Average Time Between Arrivals **0.125**

Service Rate Capacity of each server, $\mu = 11.294$ Average Service Time = **0.089**

Coefficient of Variation of Inter-arrival time, $CV(a) = 0.8212$

Coefficient of Variation of Service time, $CV(s) = 0.3747$

Basic Outputs:

The Waiting Line: Average Number Waiting in Queue (L_q) = **0.701** **<== The Approximation**

Average Waiting Time (W_q) = 0.09

Service: Average Utilization of Servers (ρ) = **70.83%**

Average Number of Customers Receiving Service = **0.70834**

The Total System (waiting line plus customers being served):

Average Number in the System (L) = **1.409**

Average Time in System (W) = **0.17615**

During this period, the estimated average waiting time will be approximately 5.4 minutes (.09 hours).

Instructions Infinite Queue Approximation Poisson Distribution

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